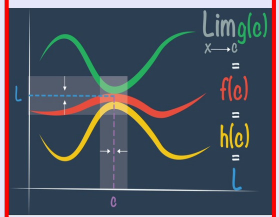
Math 261

Spring 2023

Lecture 45



Feb 19-8:47 AM

Integration with Subs.:

$$\int (2x+1)^2\,dx$$

Method I: $\int (2x+1)^2 dx = \int (4x^2+4x+1)\,dx$

$$= 4\frac{x^3}{3} + 4\frac{x^2}{2} + x + C$$

$$= \boxed{\frac{4}{3}x^3 + 2x^2 + x + C}$$

Method II: Let $u = 2x+1$

$du = 2\,dx \to \frac{du}{2} = dx$

$$\int (2x+1)^2\,dx = \int u^2 \frac{du}{2} = \frac{1}{2}\int u^2\,du$$

$$= \frac{1}{2}\cdot\frac{u^3}{3} + C$$

$$= \boxed{\frac{1}{6}(2x+1)^3 + C}$$

May 8-8:48 AM

$$\int \cos x^2 \cdot x\,dx$$

Let $u = x^2 \rightarrow du = 2x\,dx \rightarrow \frac{du}{2} = x\,dx$

$$\int \cos x^2 \cdot x\,dx = \int \cos u \,\frac{du}{2}$$

$$= \frac{1}{2}\int \cos u\,du = \frac{1}{2}\cdot \sin u + C$$

$$= \boxed{\frac{1}{2}\sin x^2 + C}$$

May 8-8:54 AM

$$\int \underbrace{(x^2+5x+8)^4 \cdot (2x+5)}\,dx$$

$u = x^2 + 5x + 8$

$du = (2x+5)dx$

$$= \int u^4\,du$$

$$= \frac{u^5}{5} + C$$

$$= \frac{1}{5}(x^2+5x+8)^5 + C$$

Verify:

$$\frac{d}{dx}\left[\frac{1}{5}(x^2+5x+8)^5 + C\right] =$$

$$\frac{1}{\cancel{5}}\cdot\cancel{5}(x^2+5x+8)^4\cdot(2x+5) + 0 =$$

$$\underbrace{(x^2+5x+8)^4\cdot(2x+5)}_{\text{Integrand}}$$

May 8-8:57 AM

$$\int \underbrace{\sin^4 x}_{u^4} \cdot \underbrace{\cos x\, dx}_{du}.$$

Let $u = \sin x \rightarrow$ Find $du = \cos x\, dx$

$$\int u^4 du = \frac{u^5}{5} + C = \boxed{\frac{1}{5}\sin^5 x + C}$$

$$\int \underbrace{\sin^4 x^2}_{u^4} \cdot \underbrace{\cos x^2 \cdot 2x\, dx}_{du}$$

Let $u = \sin x^2 \rightarrow$ Find $\frac{du}{dx} = \cos x^2 \cdot 2x$

$du = \cos x^2 \cdot 2x \cdot dx$

$$\int u^4\, du = \frac{u^5}{5} + C = \boxed{\frac{1}{5}\sin^5 x^2 + C}$$

May 8-9:02 AM

Evaluate $\int_1^2 \boxed{x^2} dx$ using Reimann Sums.

$$\int_a^b \boxed{f(x)} dx = \lim_{n \to \infty} \sum_{i=1}^{n} \boxed{f(x_i)} \cdot \Delta x \quad \text{where}$$

$$\Delta x = \frac{b-a}{n} \quad \text{and} \quad x_i = a + i\Delta x$$

$a = 1$

$b = 2$

$\Delta x = \frac{2-1}{n} = \frac{1}{n}$

$x_i = a + i\Delta x$

$= 1 + i \cdot \frac{1}{n} = 1 + \frac{i}{n}$

$$f(x_i) = f\left(1 + \frac{i}{n}\right) = \left(1 + \frac{i}{n}\right)^2 = 1 + \frac{2i}{n} + \frac{i^2}{n^2}$$

$$f(x_i) \cdot \Delta x = \left(1 + \frac{2i}{n} + \frac{i^2}{n^2}\right) \cdot \frac{1}{n} = \frac{1}{n} + \frac{2i}{n^2} + \frac{i^2}{n^3}$$

$$\sum_{i=1}^{n} f(x_i) \cdot \Delta x = \sum_{i=1}^{n} \left[\frac{1}{n} + \frac{2i}{n^2} + \frac{i^2}{n^3}\right]$$

$$= \sum_{i=1}^{n} \frac{1}{n} + \sum_{i=1}^{n} \frac{2i}{n^2} + \sum_{i=1}^{n} \frac{i^2}{n^3}$$

May 8-9:09 AM

$$= \sum_{i=1}^{n} \frac{1}{n} + \sum_{i=1}^{n} \frac{2i}{n^2} + \sum_{i=1}^{n} \frac{i^2}{n^3}$$

$$= \frac{1}{n}\cdot \sum_{i=1}^{n} 1 + \frac{2}{n^2}\sum_{i=1}^{n} i + \frac{1}{n^3}\sum_{i=1}^{n} i^2$$

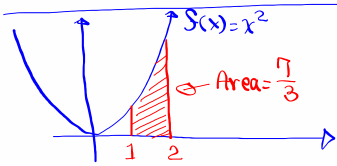
$$= \frac{1}{\cancel{n}}\cdot \cancel{n} + \frac{\cancel{2}}{n^2}\cdot \frac{n(n+1)}{\cancel{2}} + \frac{1}{n^3}\cdot \frac{n(n+1)(2n+1)}{6}$$

$$= 1 + \frac{n^2+\text{junk}}{n^2} + \frac{2n^3+\text{junk}}{6n^3}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x = \lim_{n\to\infty}\left[1 + \frac{n^2+\text{junk}}{n^2} + \frac{2n^3+\text{junk}}{6n^3}\right]$$

$$= 1 + 1 + \frac{2}{6} = 2 + \frac{1}{3} = \boxed{\frac{7}{3}} \checkmark$$

$$\int_1^2 x^2\,dx = \frac{x^3}{3}\Big]_1^2 = \frac{1}{3}\left[2^3 - 1^3\right] = \frac{1}{3}\cdot 7 = \boxed{\frac{7}{3}} \checkmark$$



May 8-9:17 AM

Evaluate $\int_{-1}^{1} x^3\,dx$ using Reimann Sums.

$f(x) = x^3$

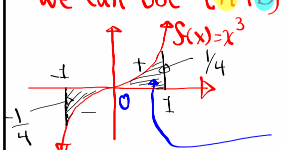
$a = -1 \to \Delta x = \frac{b-a}{n} = \frac{1-(-1)}{n} = \frac{2}{n}$

$b = 1$

$x_i = a + i\Delta x = -1 + i\cdot\frac{2}{n} = -1 + \frac{2i}{n}$

$$f(x_i)\cdot\Delta x = \left(-1 + \frac{2i}{n}\right)^3\cdot\frac{2}{n}$$

We can use $(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$



$\boxed{\text{Answer is } 0}$

$a = 0$, $b = 1$ $\Delta x = \frac{1}{n}$

$x_i = a + i\,\Delta x = \frac{i}{n}$

$$f(x_i)\cdot\Delta x = \left(\frac{i}{n}\right)^3\cdot\frac{1}{n} = \frac{1}{n^4}\,i^3$$

$$\sum_{i=1}^{n} f(x_i)\cdot\Delta x = \sum_{i=1}^{n}\frac{1}{n^4}\,i^3 = \frac{1}{n^4}\cdot\sum_{i=1}^{n} i^3 = \frac{1}{n^4}\cdot\left[\frac{n(n+1)}{2}\right]^2$$

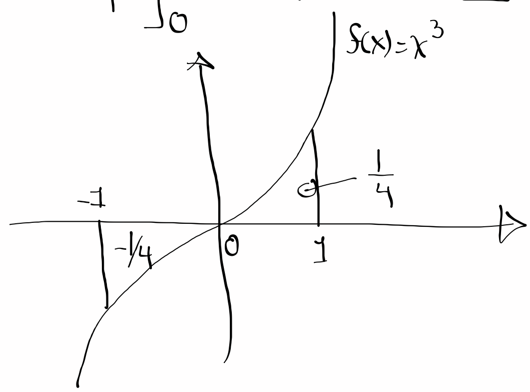
$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\cdot\Delta x = \lim_{n\to\infty}\frac{n^4+\text{junk}}{4n^4} = \boxed{\frac{1}{4}}$$

May 8-9:24 AM

$$\int_{-1}^{1} x^3\,dx = \frac{x^4}{4}\Big]_{-1}^{1} = \frac{1}{4}\left[1^4 - (-1)^4\right]$$

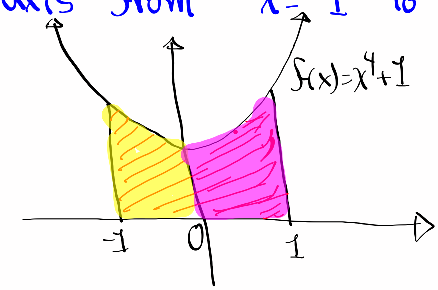
$$= \frac{1}{4}(1-1) = \boxed{0}$$

$$\int_{0}^{1} x^3\,dx = \frac{x^4}{4}\Big]_{0}^{1} = \frac{1}{4}(1^4 - 0^4) = \boxed{\frac{1}{4}}$$



May 8-9:35 AM

find the area below $f(x) = x^4+1$, above x-axis from $x=-1$ to $x=1$.



→ even Function

$f(-x) = f(x)$

Symmetric with respect to Y-axis.

$$A = \int_{-1}^{1} (x^4+1)\,dx = 2\int_{0}^{1} (x^4+1)\,dx = 2\left[\frac{x^5}{5} + x\right]\Bigg|_{0}^{1}$$

$$= 2\left[\left(\frac{1^5}{5}+1\right) - \left(\frac{0^5}{5}+0\right)\right] = 2\left(\frac{1}{5}+1\right)$$

$$= 2 \cdot \frac{6}{5} = \boxed{\frac{12}{5}}$$

May 8-9:39 AM

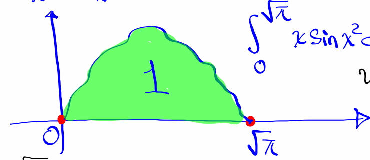
Find the area below $f(x) = x \sin x^2$, above x-axis from $x=0$ to $x=\sqrt{\pi}$.

$f(0) = 0 \cdot \sin 0^2 = 0 \cdot 0 = \boxed{0}$

$f(\sqrt{\pi}) = \sqrt{\pi} \cdot \sin(\sqrt{\pi})^2 = \sqrt{\pi} \cdot \sin \pi = \sqrt{\pi} \cdot 0 = 0$

$0 \le x \le \sqrt{\pi} \rightarrow$ QI or QII

$x \sin x^2 \rightarrow$ Positive in QI or QII.



$$\int_0^{\sqrt{\pi}} x \sin x^2 \, dx$$

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x \, dx$

$\frac{du}{2} = x \, dx$

$$\int_0^{\sqrt{\pi}} \sin x^2 \cdot x \, dx = \int_0^{\pi} \sin u \frac{du}{2}$$

$x = 0 \quad u = 0^2 = 0$

$x = \sqrt{\pi} \quad u = (\sqrt{\pi})^2 = \pi$

$$= \frac{1}{2} \cdot -\cos u \Big]_0^{\pi} = \frac{-1}{2}[\cos \pi - \cos 0]$$

$$= \frac{-1}{2}[-1 - 1]$$

$$= \frac{-1}{2}(-2) = \boxed{1}$$

May 8-9:45 AM